

Chapter 9

SQUARES AND SQUARE ROOTS (A, B, R, SQ, AND $\sqrt{}$ SCALES)

9.1 Description of the A and B scales

The A and B scales are identical scales located on the body and slide respectively. Notice that each half of the A scale consists of a complete scale similar to D; thus, the A scale is made up of two half-length D scales, one next to the other. The two sections of the scale will be referred to as "A-left" and "A-right." Similar reference will be made to the B scale.

Since one complete A scale is half the length of the complete D scale, it follows that if the hairline is moved a certain distance along the D scale, it has effectively moved twice that distance relative to the A scale. For example, if the hairline is moved to the quarter point on D, it is at the half-way point on A. Inasmuch as these distances are proportional to the logarithms of the numbers, the reading on A must correspond to the *square* of the opposite reading on D. Conversely, the reading on D represents the *square root* of the opposite reading on A.

You can easily check this relationship by taking your slide rule and moving the hairline over "2" on D. Note that this puts the hairline over "4" on A-left. Now move the hairline over "3" on D; observe that hairline is over "9" on A-left. Finally, move the hairline over "5" on D and note that "25" is the opposite reading on A-right.

Summarizing:

If the hairline is set over a number N on the **D** scale, then N^2 will be under the hairline on the **A** scale.

Conversely, if the hairline is set over a number N on the *proper half* of the **A** scale, then $\sqrt{N}$ will be under the hairline on the **D** scale.

Because the A-B scales are half-length, you cannot read them as accurately as you read the C-D scales. Note especially that there are fewer subdivisions between primary numbers on the A-B scales.

9.2 The double-length scale (R, Sq, or $\sqrt{}$)

This scale appears with different labels on various commercial slide rules; depending on the make of the rule, you may find it identified as “R” (Frederick Post Co.), “Sq” (Keuffel & Esser Co.), or “ $\sqrt{}$ ” (Pickett, Inc.) You will observe that the scale consists of two full-length sections, the first covers the same range as the left half of the D scale, whereas the second covers the same range as the right half of D. Thus, the two sections taken together represent one double-length D scale.

On the Post rule, these two sections are labeled R_1 and R_2 respectively; on the K & E rule they are denoted by Sq1 and Sq2. The Pickett model features the two sections of the $\sqrt{}$ scale “back-to-back”; that is, the two scales have a common axis, the upper markings corresponding to R_1 or Sq1, the lower markings corresponding to R_2 or Sq2. Hereafter, we shall simply use the R designation when referring to the double-length scale.

The R scale is related to the D scale in the same manner as the D scale is related to the A scale. Thus, the relationship may be described:

If the hairline is set over a number N on the **R** scale, then N^2 will be under the hairline on the **D** scale.

Conversely, if the hairline is set over a number N on the **D** scale, then $\sqrt{N}$ will be under the hairline on the *proper* section of the **R** scale.

The double-length scale may be read with greater accuracy than the A scale; hence, its advantage.

9.3 Squaring a number

Clearly, squaring a number may be handled as a multiplication in the conventional manner using the C (or CI) and D scales. However, with the A or R scales, squares may be obtained directly as illustrated in the following examples.

Example 1: $(2.48)^2 = ?$

Procedure with A scale:

1. Move HL over 248 on D.
2. Under HL read “615” on A.

Procedure with R scale:

1. Move HL over 248 on R_1 .
2. Under HL read “615” on D.

Answer is **6.15**.

Example 2: $(.00465)^2 = ?$

Write this: $(4.65 \times 10^{-3})^2 = (4.65)^2 \times 10^{-6}$.

Procedure with A scale:

1. Move HL over 465 on D.
2. Under HL read "216" on A.

Procedure with R scale:

1. Move HL over 465 on R_2 .
2. Under HL read "216" on D.

Answer is 21.6×10^{-6} or **.0000216**.

Exercise 9-1

- | | |
|-------------------|---------------------------------|
| 1. $(1.7)^2 =$ | 16. $(1.94 \times 10^2)^2 =$ |
| 2. $(5.6)^2 =$ | 17. $(2.77 \times 10^4)^2 =$ |
| 3. $(8.8)^2 =$ | 18. $(478)^2 =$ |
| 4. $(10.8)^2 =$ | 19. $(7840)^2 =$ |
| 5. $(7.4)^2 =$ | 20. $(56,000)^2 =$ |
| 6. $(3.4)^2 =$ | 21. $(162,500)^2 =$ |
| 7. $(4.27)^2 =$ | 22. $(4.05 \times 10^{-3})^2 =$ |
| 8. $(2.47)^2 =$ | 23. $(.0564)^2 =$ |
| 9. $(28.4)^2 =$ | 24. $(.00436)^2 =$ |
| 10. $(5.22)^2 =$ | 25. $(.000226)^2 =$ |
| 11. $(64.7)^2 =$ | 26. $(.0000362)^2 =$ |
| 12. $(0.405)^2 =$ | 27. $(18.65)^2 =$ |
| 13. $(13.25)^2 =$ | 28. $(56,400)^2 =$ |
| 14. $(0.144)^2 =$ | 29. $(207)^2 =$ |
| 15. $(32.2)^2 =$ | 30. $(456 \times 10^4)^2 =$ |

9.4 Finding square root of a number between 1 and 100

Example 1: $\sqrt{41.5} =$

Procedure with A scale:

Square roots are found by setting on A and reading on D. Note that the hairline may be

set over 415 on A-left or A-right. However, observe also that the desired square root is a number between 6 and 7; hence, it is clear that A-right is the proper choice.

1. Move HL over 415 on A-right.
2. Under HL read "644" on D. Answer is **6.44**.

Procedure with R scale:

Square roots are found by setting on D and reading on R_1 or R_2 . Here, we know the root is between 6 and 7; hence, answer must be on R_2 .

1. Move HL over 415 on D.
2. Under HL read "644" on R_2 . Answer is **6.44**.

Example 2: $\sqrt{5.60} =$

By inspection, root is between 2 and 3.

Procedure with A scale:

1. Move HL over 560 on A-left.
2. Under HL read "237" on D.

Answer is **2.37**.

Procedure with R scale:

1. Move HL over 560 on D.
2. Under HL read "2366" on R_1 .

Answer is **2.366**.

These examples illustrate the following rule:

1. When finding the square root of a number between 1 and 10, use **A-left** or **R_1** .
2. For numbers between 10 and 100, use **A-right** or **R_2** .

Again, you are reminded that square roots may also be obtained by reading from the proper half of the *B scale* to the *C scale*.

Verify the following:

- | | | |
|-------------------------|--------------------------|--------------------------|
| 1. $\sqrt{6.8} = 2.61$ | 4. $\sqrt{1.85} = 1.360$ | 7. $\sqrt{7.66} = 2.77$ |
| 2. $\sqrt{68} = 8.25$ | 5. $\sqrt{56.4} = 7.51$ | 8. $\sqrt{1.05} = 1.025$ |
| 3. $\sqrt{11.6} = 3.41$ | 6. $\sqrt{10} = 3.16$ | 9. $\sqrt{31.6} = 5.62$ |

9.5 A general procedure for square roots of numbers greater than 1

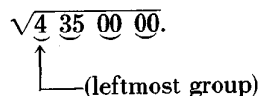
For numbers greater than 1, group the digits in *pairs* from right to left starting at the decimal point. Then:

1. If the leftmost group contains *one* digit, use **A-left** or **R₁**. If it contains *two* digits, use **A-right** or **R₂**.
2. The square root will contain the same number of *digits* to the *left* of the decimal point as there are *groups* to the *left* of the decimal point in the original number.

Example 1: $\sqrt{4,350,000} = ?$

1. Group the digits in pairs starting from the decimal point:

$$\sqrt{4 \ 35 \ 00 \ 00}.$$



 (leftmost group)

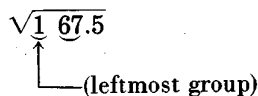
2. Leftmost group contains one digit; hence, use A-left or R₁.
3. Verify that slide rule reading is "209."
4. There are four groups to the left of the decimal point; therefore, answer will have four digits to left of decimal.

Answer is **2090**.

Example 2: $\sqrt{167.5} = ?$

1. Group the digits in pairs starting from the decimal point:

$$\sqrt{1 \ 67.5}$$



 (leftmost group)

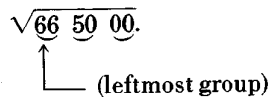
2. Leftmost group contains one digit; hence, use A-left or R₁.
3. Verify that slide rule reading is "1294."
4. There are two groups to the left of the decimal point; hence, answer will have two digits to left of decimal.

Answer is **12.94**.

Example 3: $\sqrt{665,000} = ?$

1. Group the digits in pairs:

$$\sqrt{66 \ 50 \ 00}.$$



 (leftmost group)

2. Leftmost group contains two digits; hence, use A-right or R₂.
3. Verify that answer is **815**.

Verify the following:

- | | | |
|---------------------------|------------------------------|-------------------------------|
| 1. $\sqrt{352} = 18.76$ | 4. $\sqrt{615,000} = 784$ | 7. $\sqrt{1065} = 32.6$ |
| 2. $\sqrt{2090} = 45.7$ | 5. $\sqrt{1,265,000} = 1125$ | 8. $\sqrt{27,500} = 165.8$ |
| 3. $\sqrt{173.5} = 13.17$ | 6. $\sqrt{100,000} = 316$ | 9. $\sqrt{54,500,000} = 7380$ |

Example 4: $\sqrt[4]{3170} = ?$

We use the fact that the fourth root is the square root of the square root.

1. Verify that $\sqrt{3170} = 56.3$.
2. To find the fourth root, we extract the square root again:

$$\sqrt[4]{3170} = \sqrt{\sqrt{3170}} = \sqrt{56.3} = 7.50$$

On slide rules that have both the A scale and the double-length scale, the fourth root may be read directly. Thus, on such a rule, if the hairline is moved over a number N on the A scale, then $\sqrt[4]{N}$ will be under the hairline on the proper section of R (Sq or $\sqrt{}$).

Verify the following:

- | | |
|-----------------------------|----------------------------------|
| 1. $\sqrt[4]{8.22} = 1.695$ | 4. $\sqrt[4]{16,500} = 11.35$ |
| 2. $\sqrt[4]{15.7} = 1.990$ | 5. $\sqrt[4]{5620} = 8.66$ |
| 3. $\sqrt[4]{315} = 4.21$ | 6. $\sqrt[4]{60,400,000} = 88.3$ |

9.6 General procedure for square roots of numbers less than 1

For numbers less than 1, group the digits in pairs from left to right starting at the decimal point. Then:

1. If the first nonzero group contains *one* significant digit, use **A-left** or **R₁**. If it contains *two* significant digits, use **A-right** or **R₂**.
2. For *each* zero group in the original number, the square root will have *one* zero immediately to the right of the decimal point.

Example 1: $\sqrt{.000617} = ?$

1. Group in pairs from left to right starting at decimal point:

$$\sqrt{.00 \ 06 \ 17}$$

(zero group) (first nonzero group)

2. First nonzero group contains *one* significant digit; hence, use *A-left* or R_1 . Verify that slide rule reading is "248."
3. There is *one* zero group; therefore, answer will have *one* zero immediately after the decimal point. Result is **.0248**.

Example 2: $\sqrt{.0000309} = ?$

1. Group in pairs from left to right:

$$\sqrt{.00 \ 00 \ 30 \ 90}$$

(zero groups) (first nonzero group)

2. The first nonzero group contains *two* significant digits, hence use *A-right* or R_2 . Verify that slide rule reading is "556."
3. There are *two* zero groups; therefore, answer will have *two* zeros immediately following the decimal point. Result is **.00556**.

Example 3: $\sqrt{0.426} = ?$

1. Group in pairs from left to right:

$$\sqrt{0.42 \ 60}$$

(first nonzero group)

2. First nonzero group contains *two* significant digits; hence, use *A-right* or R_2 . Verify that slide rule reading is "653."
3. There are no zero groups; therefore, answer is **0.653**.

Verify the following:

1. $\sqrt{.00475} = .0689$

5. $\sqrt{.000061} = .00781$

2. $\sqrt{0.240} = 0.490$

6. $\sqrt{.00000845} = .00291$

3. $\sqrt{.0001425} = .01194$

7. $\sqrt[3]{.0050} = 0.266$

4. $\sqrt{.0322} = 0.1794$

8. $\sqrt[3]{.0000008} = .0299$

9.7 Powers of 10 method

This method involves writing the original number in the form $M \times 10^n$, where M is a number between 1 and 100, and n is an *even* integer (positive or negative). The following examples illustrate:

Example 1: $\sqrt{430,000} = ?$

Rewrite as follows:

$$\sqrt{430,000} = \sqrt{43 \times 10^4} = \sqrt{43} \times 10^2 = \mathbf{6.56 \times 10^2}$$

Example 2: $\sqrt{.0000032} = ?$

Rewrite this:

$$\sqrt{.0000032} = \sqrt{3.2 \times 10^{-6}} = \sqrt{3.2} \times 10^{-3} = \mathbf{1.789 \times 10^{-3}}$$

Example 3: $\sqrt{2.25 \times 10^{-13}} = ?$

Rewrite:

$$\sqrt{2.25 \times 10^{-13}} = \sqrt{22.5 \times 10^{-14}} = \sqrt{22.5} \times 10^{-7} = \mathbf{4.74 \times 10^{-7}}$$

Example 4: $\sqrt{673 \times 10^7} = ?$

Rewrite this:

$$\sqrt{673 \times 10^7} = \sqrt{67.3 \times 10^8} = \sqrt{67.3} \times 10^4 = \mathbf{8.20 \times 10^4}$$

Again, you are reminded that the power of 10 must be *even*; that is, it must be *divisible by 2*.

Verify the following:

1. $\sqrt{6,100,000} = 2.47 \times 10^3$

4. $\sqrt{375 \times 10^{-9}} = 6.12 \times 10^{-4}$

2. $\sqrt{.00000055} = 7.42 \times 10^{-4}$

5. $\sqrt{14.2 \times 10^7} = 1.192 \times 10^4$

3. $\sqrt{2.75 \times 10^8} = 1.658 \times 10^4$

6. $\sqrt{10^{-5}} = 3.16 \times 10^{-3}$

Exercise 9-2

1. $\sqrt{48.6} =$

7. $\sqrt{4.78} =$

2. $\sqrt{5.43} =$

8. $\sqrt{364} =$

3. $\sqrt{12.65} =$

9. $\sqrt{3640} =$

4. $\sqrt{1.06} =$

10. $\sqrt{7460} =$

5. $\sqrt{59.8} =$

11. $\sqrt{992} =$

6. $\sqrt{10.8} =$

12. $\sqrt{19.65} =$

- | | |
|------------------------------------|------------------------------------|
| 13. $\sqrt{\pi} =$ | 32. $\sqrt{.00060} =$ |
| 14. $\sqrt{27,400} =$ | 33. $\sqrt[4]{0.861} =$ |
| 15. $\sqrt[4]{7200} =$ | 34. $\sqrt{.0070} =$ |
| 16. $\sqrt{31.4} =$ | 35. $\sqrt{3.14 \times 10^{-3}} =$ |
| 17. $\sqrt{0.764} =$ | 36. $\sqrt{.0001925} =$ |
| 18. $\sqrt{0.415} =$ | 37. $\sqrt{37,000,000} =$ |
| 19. $\sqrt{.0423} =$ | 38. $\sqrt{2.17 \times 10^{-7}} =$ |
| 20. $\sqrt{.00177} =$ | 39. $\sqrt[4]{283} =$ |
| 21. $\sqrt{.000717} =$ | 40. $\sqrt{.000211} =$ |
| 22. $\sqrt{656,000} =$ | 41. $\sqrt{2.72 \times 10^6} =$ |
| 23. $\sqrt{.0000342} =$ | 42. $\sqrt{2.18 \times 10^{-6}} =$ |
| 24. $\sqrt[4]{.00643} =$ | 43. $\sqrt{8.45} =$ |
| 25. $\sqrt[4]{68,000} =$ | 44. $\sqrt[4]{.00529} =$ |
| 26. $\sqrt{.000917} =$ | 45. $\sqrt{.000816} =$ |
| 27. $\sqrt{4.36 \times 10^6} =$ | 46. $\sqrt{2.07 \times 10^{11}} =$ |
| 28. $\sqrt{2.43 \times 10^{-4}} =$ | 47. $\sqrt{5.64 \times 10^7} =$ |
| 29. $\sqrt{5.64 \times 10^{-5}} =$ | 48. $\sqrt{28,700} =$ |
| 30. $\sqrt{27.4 \times 10^{-9}} =$ | 49. $\sqrt[4]{0.385} =$ |
| 31. $\sqrt{2,060,000} =$ | 50. $\sqrt{5.82 \times 10^5} =$ |

9.8 Formula types

Example 1: $\sqrt{3 + \frac{112}{16.2}} = ?$

1. Verify that $\frac{112}{16.2} = 6.91$.
2. Expression may now be evaluated:

$$\sqrt{3 + 6.91} = \sqrt{9.91} = 3.15.$$

As illustrated in the next examples, it is sometimes better to change the form of an expression before evaluating.

Example 2: $\pi[(4.15)^2 - (4.12)^2] = ?$

To find the difference in squares which are of about the same magnitude, more accuracy is obtained by factoring first:

$$\pi[(4.15)^2 - (4.12)^2] = \pi(4.15 + 4.12)(4.15 - 4.12) = \pi(8.27)(.03)$$

Verify that the result is **0.780**.

Example 3: $\frac{\sqrt{14} - \sqrt{13}}{2} = ?$

Here, we must find a small difference in square roots, and more accuracy is obtained if we first multiply and divide by $(\sqrt{14} + \sqrt{13})$:

$$\frac{\sqrt{14} - \sqrt{13}}{2} = \frac{(\sqrt{14} - \sqrt{13})(\sqrt{14} + \sqrt{13})}{2(\sqrt{14} + \sqrt{13})} = \frac{14 - 13}{2(\sqrt{14} + \sqrt{13})} = \frac{1}{2(\sqrt{14} + \sqrt{13})}$$

Verify that:

$$\frac{1}{2(\sqrt{14} + \sqrt{13})} = \frac{1}{2(3.74 + 3.61)} = \frac{1}{2(7.35)} = .0680.$$

Exercise 9-3

1. $\sqrt{2 + \frac{16.3}{4.2}} =$

2. $\sqrt{28 - \frac{46}{3.5}} =$

3. $\pi[(3.22)^2 - (3.19)^2] =$

4. $\pi[(236)^2 - (234)^2] =$

5. $\sqrt{\frac{1}{22} + \frac{1}{44}} =$

6. $\sqrt{\frac{1}{2.5} + \frac{1}{3.5} + \frac{1}{5.5}} =$

7. $\frac{\sqrt{230} - \sqrt{228}}{2} =$

$$8. \frac{\sqrt{31.2} - \sqrt{30.9}}{.034} =$$

$$9. \sqrt{\frac{36.7}{2.33} + \frac{165}{7.26}} =$$

$$10. \sqrt{\frac{53.7}{72.2}} - (0.72)^2 =$$

$$11. \frac{2.65}{1 - (0.985)^2} =$$

$$12. \frac{1}{\sqrt{118} - \sqrt{116}} =$$

$$13. \frac{32.2}{\sqrt{12.8} - \sqrt{12.5}} =$$

$$14. \frac{31.6 + \sqrt{153}}{12.6} =$$

$$15. \frac{.0866 - \sqrt{.00324}}{2.77} =$$

$$16. \frac{13 + \sqrt{(13)^2 + (4 \times 12 \times 21)}}{2 \times 12} =$$

$$17. \frac{-6.82 + \sqrt{(6.82)^2 - (4 \times 2.33 \times 1.52)}}{2 \times 2.33} =$$

$$18. \sqrt{(2.6)^2 + (4.7)^2 + (5.2)^2} =$$

$$19. \frac{\sqrt{17.8} - \sqrt{6.22}}{0.27 \times 3.32} =$$

$$20. \frac{\sqrt{.0535} + \sqrt{0.136}}{.0064 \times 5.22} =$$

In the following formulas, substitute as indicated and evaluate:

$$21. T = \sqrt{a + \frac{1}{a}}$$

a. $a = 1.65$

b. $a = 2.23$

c. $a = 0.644$

$$22. t = \frac{KA}{A_0} (h_1^{1/2} - h_2^{1/2})$$

a. $K = 12.6, A = 13.6, A_0 = 0.74, h_1 = 7, h_2 = 5$

b. $K = 22.8, A = 7.45, A_0 = 0.330, h_1 = 14, h_2 = 6$

c. $K = 7.25, A = 25.6, A_0 = 2.74, h_1 = 131, h_2 = 29$

$$23. R = \frac{k + \sqrt[3]{1 + r^2}}{16.1}$$

$$a. k = 18.2, r = 1.77$$

$$b. k = 5.66, r = 0.785$$

$$c. k = 34.7, r = 6.55$$

$$24. h = \frac{\sqrt{p_0} + \sqrt{p_1}}{p_0 p_1}$$

$$a. p_0 = 7.65, p_1 = 1.66$$

$$b. p_0 = 5.20, p_1 = 28.6$$

$$c. p_0 = .0440, p_1 = 0.655$$

$$25. d = \sqrt{a^2 + b^2 + c^2}$$

$$a. a = 2.2, b = 3.6, c = 4.7$$

$$b. a = 13.7, b = 2.27, c = 8.66$$

9.9 The quadratic formula

The two roots of the quadratic equation $ax^2 + bx + c = 0$ are given by the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example: Solve for x : $2.84x^2 + 5.35x - 3.75 = 0$.

1. In this case, $a = 2.84$, $b = 5.35$, and $c = -3.75$. Substituting these values, the formula becomes:

$$x = \frac{-5.35 \pm \sqrt{(5.35)^2 - 4(2.84)(-3.75)}}{2(2.84)}$$

$$2. \text{ Verify that } x = \frac{-5.35 \pm \sqrt{28.6 + 42.5}}{5.68} = \frac{-5.35 \pm 8.44}{5.68}$$

3. Finally verify that the two roots are **0.544** and **-2.43**.

Exercise 9-4

Solve for x :

$$1. 2x^2 + 3x - 1 = 0$$

$$5. x^2 - 11.7x + 4.55 = 0$$

$$2. 5x^2 - 17x + 6 = 0$$

$$6. 2.65x^2 + 6.82x + 1.70 = 0$$

$$3. 7x^2 - 13x - 5 = 0$$

$$7. 31.2x^2 + 23.7x + 2.18 = 0$$

$$4. 16x^2 + x - 0.30 = 0$$

$$8. 0.810x^2 + 17.6x - 31.2 = 0$$

Chapter 10

CUBES AND CUBE ROOTS (K AND $\sqrt[3]{}$ SCALES)

10.1 Description of the K scale*

The K scale consists of three complete scales, one next to the other, each similar to the D scale. Thus, each section represents a one-third-length D scale. We shall refer to these sections as "K-left," "K-middle," and "K-right." Because of the logarithmic nature of the scales, it follows that the one-third-length K scale leads to a cube-cube root relationship with the D scale. To verify this, suppose you now take your slide rule and move the hairline over "2" on the D scale. Note that this positions the hairline over "8" on K-left. Now move the hairline over "3" on D; note that this puts the hairline over "27" on K-middle. Finally, move the hairline over "5" on D and observe that the hairline is over "125" on K-right. In each case, the reading on K is the *cube* of the corresponding reading on D.

Summarizing:

If the hairline is set over a number N on the **D** scale, then N^3 will be under the hairline on the **K** scale.

Conversely, if the hairline is set over a number N on the *proper section* of the **K** scale, then $\sqrt[3]{N}$ will be under the hairline on the **D** scale.

In making settings and readings on the K scale, carefully note how the primary intervals are subdivided. Clearly, the use of this scale involves considerable loss in accuracy; over much of the scale you will find yourself straining to approximate the third digit.

*If your slide rule does not have a K scale, but does have a scale labeled $\sqrt[3]{}$, then refer to section 10.7 of this chapter.

10.2 Cubing a number using the K scale

Of course, cubing a number may be treated as successive multiplication using the C, CI, and D scales, thus retaining the accuracy of these scales. However, as indicated in the previous section, cubes may be read directly from D to K.

Example 1: $(3.45)^3 = ?$

1. Move HL over 345 on D.
2. Under HL read "410" on K-middle. Answer is **41.0**.

Example 2: $(645)^3 = ?$

Write this: $(6.45 \times 10^2)^3 = (6.45)^3 \times 10^6$.

1. Move HL over 645 on D.
2. Under HL read "268" on K-right.

Answer is 268×10^6 or **2.68×10^8** .

Example 3: $(.00172)^3 = ?$

Write this: $(1.72 \times 10^{-3})^3 = (1.72)^3 \times 10^{-9}$.

1. Move HL over 172 on D.
2. Under HL read "509" on K-left. Answer is **5.09×10^{-9}** .

Exercise 10-1

- | | |
|-------------------|------------------------------|
| 1. $(1.6)^3 =$ | 11. $(0.512)^3 =$ |
| 2. $(2.4)^3 =$ | 12. $(10.65)^3 =$ |
| 3. $(6.5)^3 =$ | 13. $(12.7)^3 =$ |
| 4. $(7.2)^3 =$ | 14. $(8180)^3 =$ |
| 5. $(1.23)^3 =$ | 15. $(29,600)^3 =$ |
| 6. $(5.82)^3 =$ | 16. $(364)^3 =$ |
| 7. $(3.24)^3 =$ | 17. $(135)^3 =$ |
| 8. $(2.76)^3 =$ | 18. $(0.907)^3 =$ |
| 9. $(8.04)^3 =$ | 19. $(1265)^3 =$ |
| 10. $(0.764)^3 =$ | 20. $(4.65 \times 10^4)^3 =$ |

21. $(0.787)^3 =$

26. $(.00284)^3 =$

22. $(0.215)^3 =$

27. $(.00847)^3 =$

23. $(3.96 \times 10^{-3})^3 =$

28. $(17,550)^3 =$

24. $(0.404)^3 =$

29. $(68.2)^3 =$

25. $(.0643)^3 =$

30. $(.000545)^3 =$

10.3 Finding cube root of a number between 1 and 1000

As stated before, if the hairline is set over a number on the proper section of the K scale, the cube root of that number will be under the hairline on the D scale. You can easily verify the following rule for selecting the proper section of the K scale:

1. For cube roots of numbers between *1 and 10*, use ***K-left***.
2. For numbers between *10 and 100*, use ***K-middle***.
3. For numbers between *100 and 1000*, use ***K-right***.

Example 1: $\sqrt[3]{3.60} = ?$

1. The number is between 1 and 10; hence, move HL over 360 on K-left.
2. Under HL read "1532" on D. Answer is **1.532**.

Example 2: $\sqrt[3]{45.4} = ?$

1. The number is between 10 and 100; hence, move HL over 454 on K-middle.
2. Under HL read "357" on D. Answer is **3.57**.

Example 3: $\sqrt[3]{720} = ?$

1. The number is between 100 and 1000; hence, move HL over 720 on K-right.
2. Under HL read "896" on D. Answer is **8.96**.

Verify the following:

1. $\sqrt[3]{7.4} = 1.950$

2. $\sqrt[3]{32} = 3.17$

3. $\sqrt[3]{615} = 8.50$

4. $\sqrt[3]{12.1} = 2.30$

6. $\sqrt[3]{10} = 2.15$

8. $\sqrt[3]{75} = 4.22$

5. $\sqrt[3]{4.25} = 1.620$

7. $\sqrt[3]{100} = 4.64$

9. $\sqrt[3]{134} = 5.12$

10.4 General procedure for cube roots

In order to determine which section of the K scale to use for any number, the digits may be grouped in a manner similar to that used for square roots. However, instead of marking the digits off in pairs, they are marked off in groups of *three*. The following rule then applies:

For numbers greater than 1:

1. If the leftmost group contains *one* digit, use **K-left**; if it contains *two* digits, use **K-middle**; if it contains *three* digits, use **K-right**.
2. The cube root will contain the same number of *digits to the left* of the decimal point as there are *groups to the left* of the decimal point in the original number.

For numbers less than 1:

1. If the first nonzero group contains *one* significant digit, use **K-left**; if it contains *two* significant digits, use **K-middle**; if it contains *three* significant digits, use **K-right**.
2. For *each zero group* in the original number, the cube root will contain *one zero* between the decimal point and the first significant figure.

Example 1: $\sqrt[3]{43,500} = ?$

1. Grouping the digits: $\sqrt[3]{\overline{43} \overline{500}}$
2. Leftmost group contains *two* digits; hence, use **K-middle**. Verify that the slide rule reading is "352."
3. There are *two* groups; hence, answer contains *two* digits to the left of the decimal point. Result is **35.2**.

Example 2: $\sqrt[3]{2,140,000} = ?$

1. Grouping the digits: $\sqrt[3]{\overline{2} \overline{140} \overline{000}}$
2. Leftmost group contains *one* digit; hence, use **K-left**. Verify that slide rule reading is "1288."
3. There are *three* groups; therefore, answer is **128.8**.

Example 3: $\sqrt[3]{.000720} = ?$

1. Grouping the digits: $\sqrt[3]{.000\ 720}$
2. First nonzero group contains *three* significant digits; hence, use *K-right*. Slide rule reading is "896."
3. There is *one zero group*; therefore, there is *one zero* between the decimal point and the first significant digit. Answer is **.0896**.

Example 4: $\sqrt[3]{.00564} = ?$

1. Grouping the digits: $\sqrt[3]{.005\ 640}$
2. First nonzero group contains *one* significant digit; hence, use *K-left*. Slide rule reading is "178."
3. There is *no zero group*; therefore, answer is **0.178**.

Verify the following:

- | | |
|--------------------------------|-----------------------------------|
| 1. $\sqrt[3]{6400} = 18.57$ | 6. $\sqrt[3]{.0000048} = .01687$ |
| 2. $\sqrt[3]{.0215} = 0.278$ | 7. $\sqrt[3]{62,000,000} = 396$ |
| 3. $\sqrt[3]{365,000} = 71.5$ | 8. $\sqrt[3]{.00000075} = .00909$ |
| 4. $\sqrt[3]{0.820} = 0.936$ | 9. $\sqrt[3]{4,250,000} = 162$ |
| 5. $\sqrt[3]{.000100} = .0464$ | 10. $\sqrt[3]{.0325} = 0.319$ |

10.5 Powers of 10 method

Using this method, the original number is written in the form $M \times 10^n$, where M is a number between 1 and 1000, and n is a positive or negative integer which is *divisible by three*. The following examples illustrate:

Example 1: $\sqrt[3]{56,100,000} = ?$

Rewrite: $\sqrt[3]{56.1 \times 10^6} = \sqrt[3]{56.1} \times 10^2 = 3.83 \times 10^2 = \mathbf{383}$.

Example 2: $\sqrt[3]{.000000750} = ?$

Rewrite: $\sqrt[3]{750 \times 10^{-9}} = \sqrt[3]{750} \times 10^{-3} = 9.09 \times 10^{-3} = \mathbf{.00909}$.

Example 3: $\sqrt[3]{4.66 \times 10^{10}} = ?$

Rewrite: $\sqrt[3]{46.6 \times 10^9} = \sqrt[3]{46.6} \times 10^3 = 3.60 \times 10^3 = \mathbf{3600}$.

Example 4: $\sqrt[3]{2.34 \times 10^{-10}} = ?$

Rewrite: $\sqrt[3]{234 \times 10^{-12}} = \sqrt[3]{234} \times 10^{-4} = 6.16 \times 10^{-4} = .000616$.

Again, you are reminded that the power of 10 must be *divisible by three*.

Verify the following:

1. $\sqrt[3]{1.84 \times 10^7} = 264$

5. $\sqrt[3]{482 \times 10^{16}} = 1.69 \times 10^6$

2. $\sqrt[3]{6.47 \times 10^{-13}} = 8.65 \times 10^{-5}$

6. $\sqrt[3]{42 \times 10^{-7}} = .01613$

3. $\sqrt[3]{10^8} = 464$

7. $\sqrt[3]{36.5 \times 10^8} = 1540$

4. $\sqrt[3]{10^{-11}} = .000216$

8. $\sqrt[3]{7.25 \times 10^{11}} = 8980$

Exercise 10-2

1. $\sqrt[3]{7.43} =$

16. $\sqrt[3]{0.912} =$

2. $\sqrt[3]{8.22} =$

17. $\sqrt[3]{.0207} =$

3. $\sqrt[3]{10.9} =$

18. $\sqrt[3]{.0611} =$

4. $\sqrt[3]{96.2} =$

19. $\sqrt[3]{.0829} =$

5. $\sqrt[3]{17.65} =$

20. $\sqrt[3]{.00299} =$

6. $\sqrt[3]{124} =$

21. $\sqrt[3]{.00748} =$

7. $\sqrt[3]{742} =$

22. $\sqrt[3]{3920} =$

8. $\sqrt[3]{2.94} =$

23. $\sqrt[3]{4680} =$

9. $\sqrt[3]{294} =$

24. $\sqrt[3]{9600} =$

10. $\sqrt[3]{57.4} =$

25. $\sqrt[3]{2000} =$

11. $\sqrt[3]{1.11} =$

26. $\sqrt[3]{7640} =$

12. $\sqrt[3]{11.1} =$

27. $\sqrt[3]{10,700} =$

13. $\sqrt[3]{111} =$

28. $\sqrt[3]{23,600} =$

14. $\sqrt[3]{.0624} =$

29. $\sqrt[3]{64,300} =$

15. $\sqrt[3]{0.526} =$

30. $\sqrt[3]{575,000} =$

31. $\sqrt[3]{207,000} =$

41. $\sqrt[3]{2.76 \times 10^{-6}} =$

32. $\sqrt[3]{87,500} =$

42. $\sqrt[3]{4.68 \times 10^{-5}} =$

33. $\sqrt[3]{.00726} =$

43. $\sqrt[3]{5.64 \times 10^{-2}} =$

34. $\sqrt[3]{.000276} =$

44. $\sqrt[3]{11.6 \times 10^{-4}} =$

35. $\sqrt[3]{.0000427} =$

45. $\sqrt[3]{1.95 \times 10^5} =$

36. $\sqrt[3]{23,600,000} =$

46. $\sqrt[3]{12.65 \times 10^8} =$

37. $\sqrt[3]{.00000717} =$

47. $\sqrt[3]{0.416 \times 10^{10}} =$

38. $\sqrt[3]{180,000,000} =$

48. $\sqrt[3]{4.26 \times 10^{-10}} =$

39. $\sqrt[3]{.0000007} =$

49. $\sqrt[3]{7.61 \times 10^{14}} =$

40. $\sqrt[3]{76.1 \times 10^6} =$

50. $\sqrt[3]{3.37 \times 10^{-11}} =$

10.6 Finding $(N)^{3/2}$ and $(N)^{2/3}$

The general problem of raising a number to any power is discussed later in connection with the log-log scales. However, certain fractional powers may be read directly using the A and K scales.

Example 1: $(56.2)^{3/2} = (\sqrt{56.2})^3 = ?$

1. Move HL over 562 on A-right. Note that $\sqrt{56.2}$ is now under HL on D, and $(\sqrt{56.2})^3$ is under IIL on K.
2. Under HL read "421" on K. Answer is **421**.

If you do not have the A scale, you may first evaluate $(56.2)^3$, then find the square root of this on R.

Example 2: $(7.2)^{2/3} = (\sqrt[3]{7.2})^2 = ?$

1. Move HL over 72 on K-left. Observe that $\sqrt[3]{7.2}$ is now under HL on D, and $(\sqrt[3]{7.2})^2$ is under HL on A.
2. Under HL read "373" on A. Answer is **3.73**.

If no A scale is present, you may first find $\sqrt[3]{7.2}$, then multiply this by itself.

These examples illustrate the following relationship:

1. If HL is set over a number N on the **A** scale, then $(N)^{3/2}$ will be under HL on the **K** scale.
2. If HL is set over a number N on the **K** scale, then $(N)^{2/3}$ will be under HL on the **A** scale.

Example 3: $(2.75 \times 10^{13})^{2/3} = ?$

Write this: $(27.5 \times 10^{12})^{2/3} = (27.5)^{2/3} \times 10^8$.

1. Move HL over 275 on K-middle.
2. Under HL read "910" on A. Answer is **9.10×10^8** .

Exercise 10-3

- | | |
|------------------------|-------------------------------------|
| 1. $(4.5)^{3/2} =$ | 11. $(342)^{2/3} =$ |
| 2. $(37)^{3/2} =$ | 12. $(1.65)^{2/3} =$ |
| 3. $(16.3)^{3/2} =$ | 13. $(2340)^{2/3} =$ |
| 4. $(0.814)^{3/2} =$ | 14. $(.0621)^{2/3} =$ |
| 5. $(.0614)^{3/2} =$ | 15. $(.000175)^{2/3} =$ |
| 6. $(.00743)^{3/2} =$ | 16. $(0.445)^{3/2} =$ |
| 7. $(374)^{3/2} =$ | 17. $(3.22 \times 10^{-9})^{3/2} =$ |
| 8. $(.000514)^{3/2} =$ | 18. $(7.45 \times 10^8)^{2/3} =$ |
| 9. $(2460)^{3/2} =$ | 19. $(63 \times 10^{-7})^{2/3} =$ |
| 10. $(46.2)^{2/3} =$ | 20. $(5.75 \times 10^9)^{3/2} =$ |

10.7 The triple-length scale ($\sqrt[3]{}$)

Some Pickett models feature a $\sqrt[3]{}$ scale consisting of three full-length sections; the first covers the same range as the first third of the regular D scale (left index to "2154"), the second covers the range of the middle third of D ("2154" to "464"), and the third corresponds to the final third of the D scale ("464" to right index). We shall refer to these sections as $(\sqrt[3]{})_1$, $(\sqrt[3]{})_2$, and $(\sqrt[3]{})_3$ respectively. Clearly, the three sections taken together represent one triple-length D scale.

The $\sqrt[3]{}$ scale is related to the D scale as follows:

If the hairline is set over a number N on the $\sqrt[3]{}$ scale, then N^3 will be under the hairline on the **D** scale.

Conversely, if the hairline is set over a number N on the **D** scale, then $\sqrt[3]{N}$ will be under the hairline on the *proper section* of the $\sqrt[3]{}$ scale.

Example 1: $(3.47)^3 = ?$

1. Move HL over 347 on $(\sqrt[3]{})_2$.
2. Under HL read "418" on D. Answer is **41.8**.

Example 2: $\sqrt[3]{6450} = ?$

To determine which section of the $\sqrt[3]{}$ scale applies, you should refer to sections 10.3 and 10.4 of this chapter—substituting $(\sqrt[3]{})_1$ for K-left, $(\sqrt[3]{})_2$ for K-middle, and $(\sqrt[3]{})_3$ for K-right.

1. Grouping the digits: $\sqrt[3]{6\ 450}$.
2. Leftmost group contains *1 digit*; hence, answer is on $(\sqrt[3]{})_1$.
3. Move HL over 645 on D. Under HL read "1861" on $(\sqrt[3]{})_1$.
4. There are *two groups*; hence, there are *two digits* to the left of the decimal point in the answer. Result is **18.61**.

Example 3: $\sqrt[3]{.000224} = ?$

1. Grouping the digits: $\sqrt[3]{.000\ 224}$.
2. The first nonzero group contains *three* significant digits, hence answer is on $(\sqrt[3]{})_3$.
3. Move HL over 224 on D. Under HL read "607" on $(\sqrt[3]{})_3$.
4. There is *one zero group*; therefore, answer has *one zero* immediately following the decimal point. Result is **.0607**.

Exercises 10-1 and 10-2 may be used for more drill with these scales.